L2 - Kirchoff's Law & Thevenin's Theorem

PHYS 301: Analog and Digital Electronics

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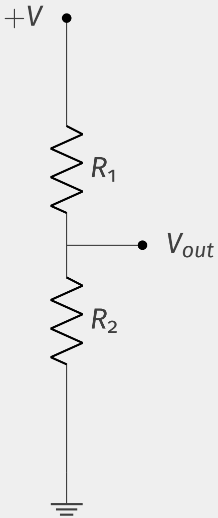
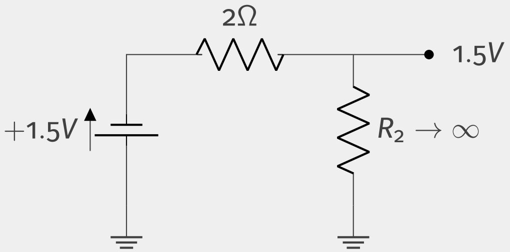


$$V_{out} = \frac{R_2}{R_1 + R_2} V$$

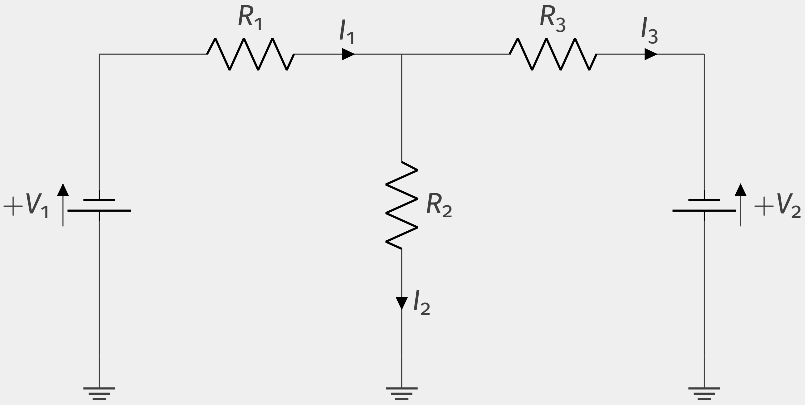
Open Circuit Voltage



$$V_{out} = \lim_{R_2 \to \infty} \frac{R_2}{R_1 + R_2} 1.5V \approx 1.5V \frac{\infty}{\infty}$$

$$\frac{1}{1 + \frac{R_1}{R_2}} 1.5V \approx \frac{1}{1+0} 1.5V = 1.5V$$

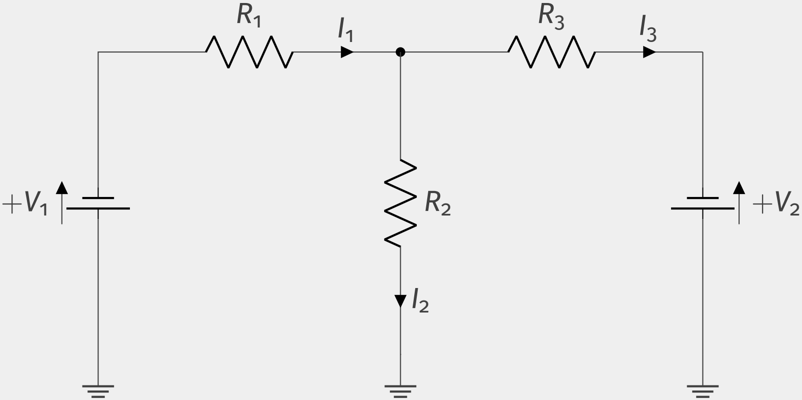
Kirchoff's Laws



Goal: Solve for I_1, I_2 and I_3

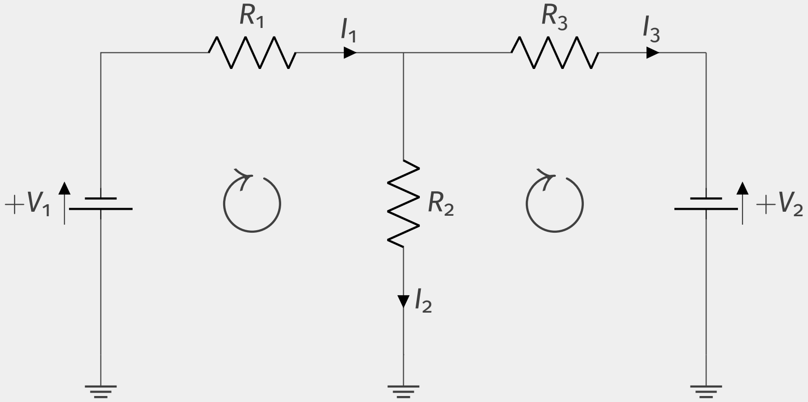
Ohm's Law doesn't provide enough information.

Junction Rule



$$\sum I_{in} = \sum I_{out}$$

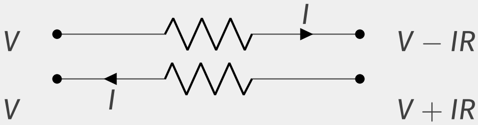
Loop Rule



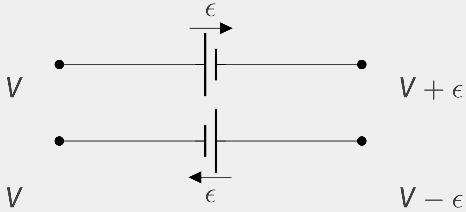
$$\sum_{\text{loop}} \Delta V = 0$$

Loop Rule & Current Direction

Components



Power Supplies



Step 1: Set up junction and loop rule equations.

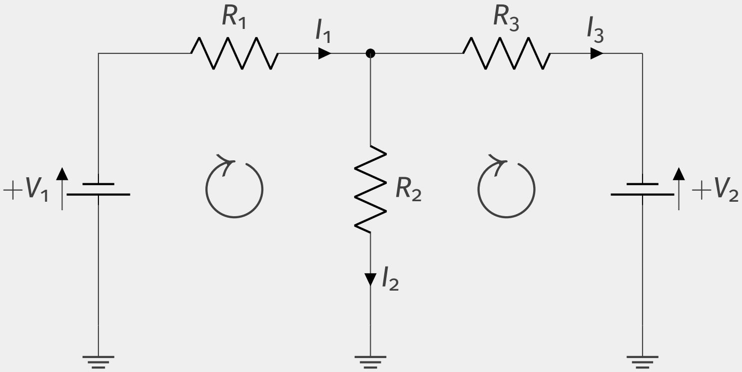
$$I_1 = I_2 + I_3 \ (1)$$

$$V_1 - I_1 R_1 - I_2 R_2 = 0 \ (2)$$

$$0 + I_2 R_2 - I_3 R_3 - V_2 = 0 \ (3)$$

Step 2: Solve system of linear equations.

Example Problem



$V_1 = 15\text{V}$, $V_2 = 10\text{V}$
$R_1 = 4\Omega$, $R_2 = 5\Omega$, $R_3 = 2\Omega$

Example Problem Cont.

$$I_1 = I_2 + I_3$$

$$0 + 15V - 4I_1 - 5I_2 = 0$$

$$0 + 5I_2 - 2I_3 - 10V = 0$$

$$15 = 4I_1 + 5I_2$$

$$10 = 5I_1 + 2I_3$$

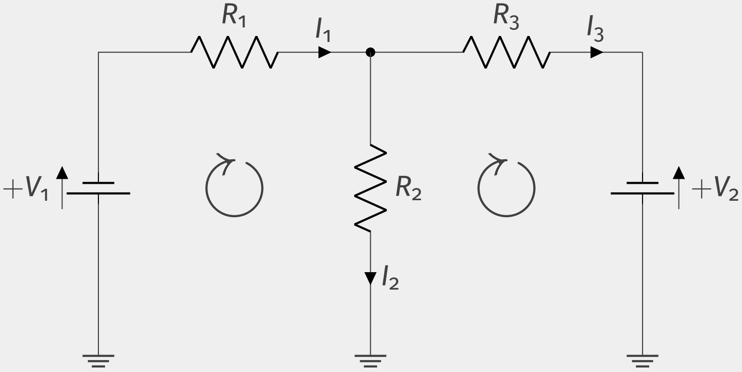
$$15 = 4I_2 + 4I_3 + 5I_2 = 9I_2 + 4I_3$$

$$+2 \times [10 = 5I_2 - 2I_3]$$

$$35 = 19I_2$$

$$I_3 = \frac{5}{2}(\frac{35}{19}) - 5 = \frac{-15}{38}A$$

$$I_1 = \frac{35}{19} - \frac{15}{38} = \frac{55}{38}A$$

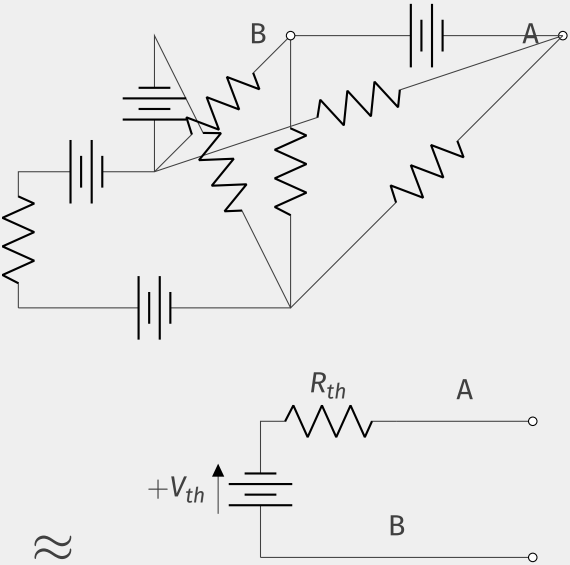


$$V_{junction} = 5I_2 = 5\frac{35}{19} = \frac{175}{19}V \approx 9.2V$$

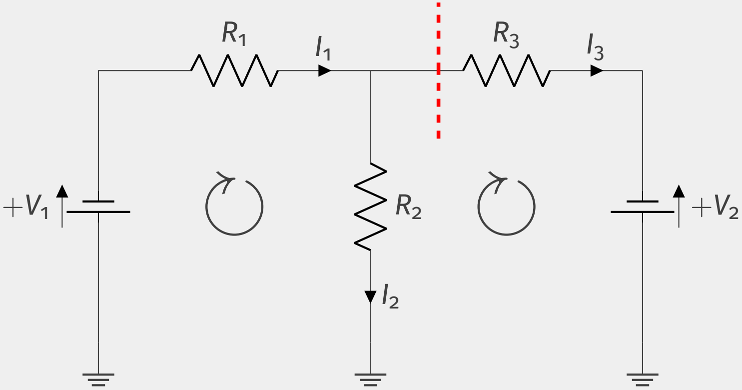
Wolfram Solver:
https://www.wolframalpha.com/input/?i=I_1%3DI_2%2BI_3,+15%3D4I_1%2B5I_2,+10%3D5I_2-2I_3

Thevenin's Theorem

Complex Circuit

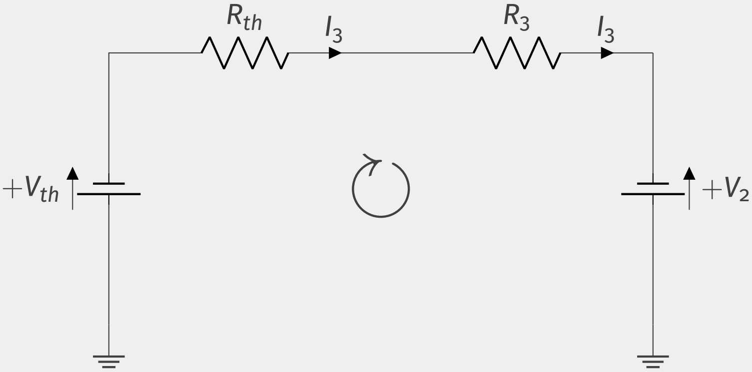


Example



$$V_1 = 15\text{V},\ V_2 = 10\text{V}$$
$$R_1 = 4\Omega,\ R_2 = 5\Omega,\ R_3 = 2\Omega$$

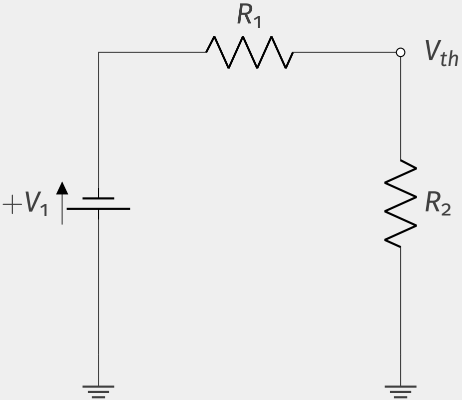
Simplified Circuit



$$I_3 = \frac{V_{th} - V_2}{R_{th} + R_3}$$

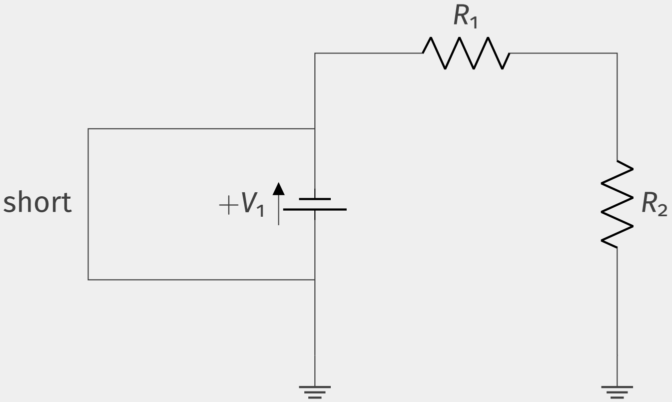
$$V_{th} - I_3 R_{th} - I_3 R_3 - V_2 = 0$$

Finding V_{th}



$$V_{th} = \frac{R_2}{R_1 + R_2} V_1 = \frac{5}{4+5} 15V = \frac{25}{3} V$$

Finding R_{th}



$$\frac{1}{R_{th}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{4\Omega} + \frac{1}{5\Omega} = \frac{9}{20\Omega}$$

Current

$$I_3 = \frac{V_{th} - V_2}{R_{th} + R_3} = \frac{\frac{25}{3} - 10}{\frac{20}{9} + 2} = \frac{-15}{38}\text{A}$$

Junction Voltage

$$V_{R_3} = I_3 R_3 = \frac{-15}{38} \times 2 = \frac{-15}{19}\text{V}$$

$$V_{junction} = 10 - V_{R_3} = \frac{175}{19}$$