Instructor Notes – Isaac Woodard 11 December 2020
PHYS 111 Lab – Archimedes' Principle

# Overview

The goal of today's lab is to measure the density of several objects using three different methods: geometrical analysis, water displacement and Archimedes' principle. In the end, the pros and cons of each method will be compared.

# Theory

Let's start with a definition of density.

$$\rho = \frac{m}{V}$$

This definition generally assumes the density of the object is uniform, but it is effectively always true if we only care about the object as a whole and aren't worried about examining different pieces of it.

Typically, we'd measure the object's mass using a scale, but as a lead-up to using Archimedes' principle, we're instead going to measure the object's weight by hanging it from a force probe.

$$W = \rho_0 V g$$

With this equation, what we need to know to calculate the object's density is its weight and its volume.

### Geometrical Analysis

The first method we'll use to measure density is geometrical analysis. In this method, the density is found by first measuring the object's weight and then taking some length measurements to calculate its volume geometrically.

This method assumes the object is perfectly geometric and is difficult to apply to irregularly shaped objects.

### Water Displacement

The second method addresses the issue with irregularly shaped objects by measuring the volume with water displacement. Instead of measuring the object with geometry, it is placed in a container of water and the change in volume of the container is measured.

Measuring the change in volume is quickest with a graduated cylinder, but there are many cases where the object might not fit into a graduated cylinder. For large objects, it is easier to use a container with a spout in its side and then fill the container with water up to the spout. Placing the object in the container will cause the water to overflow through the spout. Collecting the overflowed water in another container gives an easy way to measure the volume of the object because the volume of the overflowed water will be equal to the volume of the object.

The main disadvantages of this method are the amount of setup required and the amount of measurement error involved with measuring the volume of small objects.

### Archimedes' Principle

The third method utilizes Archimedes' principle. Archimedes' principle is an explanation of the buoyant force an object experiences when submerged in a liquid. When an object is placed in a container of water, the object has to push some of the water out of its way to make room for itself. The water the object pushes aside, though, pushes back on the object. This pushback is the buoyant force.

$$Diagram: Bucket\ of\ Water\ \&\ Object$$

The amount of water that is pushed out of the way depends on the density of the object. If the object is less dense than water, it won't sink and only part of the object will be submerged. If the object is denser than water, though, it will sink. In this case, the amount of water the object must push out of the way is equal to the object's volume.

$$Diagram: Sinking\ \&\ Floating\ Object$$

The buoyant force was used by Archimedes thousands of years ago to measure the density of objects. Archimedes discovered that the buoyant force is equal to the weight of the water an object must push out of its way.

$$Diagram: Force\ Diagram\ for\ Buoyant\ Force$$

If we draw a force diagram for a submerged object there are two forces acting on the object: the object's weight and the buoyant force. The object's weight is given by

$$W = \rho_0 V g \quad (W_a)$$

and the buoyant force is given by

$$B = \rho_f V g \quad (W_f)$$

From these expressions we can derive an equation for the density of the submerged object.

$$\rho_0 = \frac{\rho_W}{1 - \frac{W_w}{W_a}}$$

## Procedure

Densities will be measured for a total of four objects. Two of those objects should be geometrical and two should be irregular. This means the densities of the first two objects should be measured with all three methods, while the densities of the last two objects should only be measured with the last two methods.

After calculating the densities, look up values online for the densities of the materials. Use these to calculate percent errors for the measured density values.